A COMPENSATION METHOD OF MEASURING THE THERMAL ACTIVITY OF LIQUIDS

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A compensation method of measuring the thermal activity of liquids is outlined in which a low-inertia temperature probe is heated by a current with both an alternating and a direct component.

A method of measuring the thermal activity of liquids has been described in [1, 2, 3] where low-inertia temperature probes (a foil, a filament) are heated by alternating current. The gist of this method is that the electrical resistance of a low-inertia temperature probe heated by alternating current fluctuates at double the current frequency. Such a periodic variation of the resistance in an ac current results in the appearance of a triple-frequency current component. The amplitude of current fluctuation is determined by the thermal activity coefficient of the medium.

The authors have developed another method of measuring the thermal activity of liquids, based on heating the temperature probe by a current which has both an alternating and a direct component. The thermal activity of the medium is determined from the magnitude of the capacitance necessary to compensate the probe reactance on a Wheatstone bridge. The advantages of this other method are the simple compensation technique of measurements and that no alternating voltages have to be measured for determining the thermal activity. Furthermore, synchronous detection can be applied here for accurate compensation of the useful signals picked off the output diagonal of the bridge, since the modulating voltage itself can serve as the reference signal for the synchronous detector, while a frequency tripler is required in the plain ac heating method [4]. Thus, the use of the compensation circuit together with a narrow-band amplifier and with synchronous detection makes it possible to attain a relatively high sensitivity to fluctuations in the thermal activity of liquids at temperature pulsation amplitudes of the order of 0.1-0.3°C.

The basic relations between the thermal activity of a liquid, the amplitude of probe temperature pulsations, and the phase shift have been derived earlier in [2, 3]. The character of temperature pulsations in a wire (diameter $2r_0$) pulled through a narrow capillary was analyzed assuming that 1) heat is transmitted radially and by conduction only, which is characteristic of long wires ($l/r_0 \gg 1$), and 2) the temperature of the entire wire is the same at every instant of time, which is valid for thin wires with relatively slow temperature fluctuations.

On the basis of these assumptions, solving the equation of heat conduction will yield an expression for the temperature field profile around the wire:

$$T = \Theta \frac{H_0^{(1)}(z)}{H_0^{(1)}(z_0)} e^{i\omega t} + \tau(r).$$

Then, setting up and solving the equation of heat balance, we find the following relations for the amplitude of temperature pulsations and its phase shift with respect to power:

$$\frac{|\Theta|}{\Theta_0} = \left[1 + 4\frac{\varkappa}{b} \cdot \frac{h_1}{h_0} \sin\left(\eta_0 - \eta_1 - \frac{\pi}{4}\right) + 4\left(\frac{\varkappa}{b}\right)^2 \left(\frac{h_1}{h_0}\right)^2\right]^{-1/2},$$

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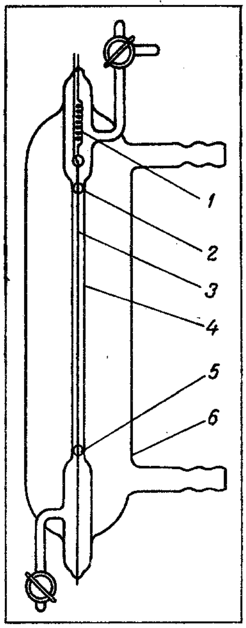

Fig. 1

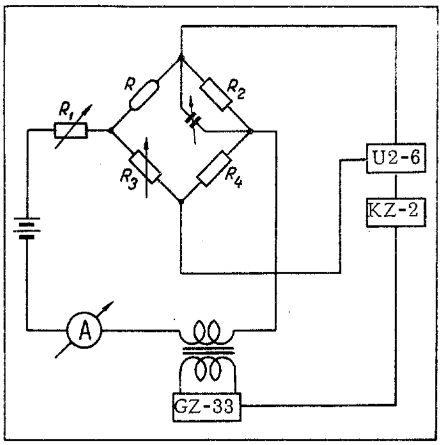

Fig. 2

Fig. 1. Instrument for measuring the thermal activity of liquids: 1) tungsten spring, 2) and 5) centering beads, 3) tungsten filament, 4) instrument capillary, 6) thermostat jacket.

Fig. 2. Compensation circuit with a variable capacitance: R is the probe resistance, R_1 is the R58 resistor bank resistance, R_2 is the reference resistance, R_3 is the adjustable resistance, and R_4 is the fixed resistance.

$$\operatorname{tg}\varphi=\frac{1+2\,\dfrac{h_1}{h_0}\cdot\dfrac{\varkappa}{b}\sin\left(\eta_0-\eta_1-\dfrac{\pi}{4}\right)}{2\,\dfrac{h_1}{h_0}\cdot\dfrac{\varkappa}{b}\cos\left(\eta_0-\eta_1-\dfrac{\pi}{4}\right)}. \tag{2}$$

Test Apparatus and Measurement Procedure. The measuring instrument (Fig. 1) consists of a capillary 1 mm in diameter with a tungsten filament probe 0.020 mm in diameter pulled through it. Glass beads ensure that the probe is properly centered inside, with a tungsten spring providing the necessary tension along the tube. A TS-16 thermostat is used here. This construction of the apparatus ensures that the liquid will reach a thermal equilibrium quickly, it prevents air bubbles from entering the capillary tube while the latter is filled, and it requires only a small quantity of tested liquid (approximately 1 cm^3).

The electrical circuit of the apparatus is shown in Fig. 2. The probe is connected into an arm of the Wheatstone bridge consisting, besides, of noninductive resistor banks R517M and R58, a reference resistance R_2, and a variable capacitance R513. All lead wires are shielded here.

The bridge is supplied from heater-voltage batteries and an acoustic generator GZ-33. The current energizes the bridge circuit is measured with a Class 0.5 ammeter. A KZ-2 synchronous detector and a U2-6 amplifier are used for precise balancing. The reference signal of the synchronous detector makes it possible to raise the accuracy of thermal activity measurements at low amplitudes of probe temperature pulsations (0.1-0.3°C) during the test. Such an arrangement makes the synchronous detector almost insensitive to amplitude fluctuations in the acoustic generator voltage. This improves considerably the balancing of the compensation circuit, since the fundamental signal of bridge unbalance can be extracted here without noise and then be compensated by adjusting the amplifier gain and the phase shift. In our experiment the exact generator frequencies were determined with a PS-100 pulse counter and a stop watch. The frequency stability of the generator was adequate for our purposes. Measurements were made in the 20-400 Hz range. They showed that a modulation frequency within this range does not affect the results.

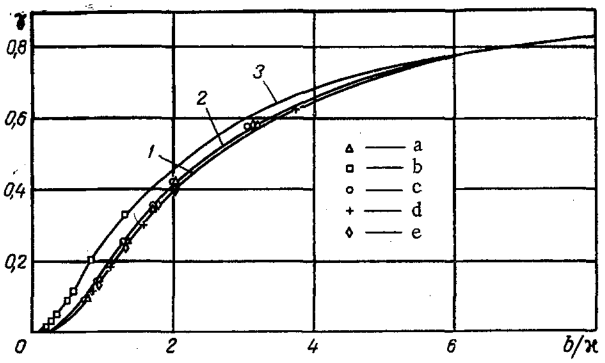

Fig. 3. Comparison between experimental and theoretical values of the thermal activity of liquids with $c_p\rho/c'_p\rho' = 0.5$ (1), 0.6 (2), and 1.52 (3): n-hexane (a), water (b), n-heptane (c), carbon tetrachloride (d), and toluene (e).

There is an upper limit to allowable test frequencies. This is because the probe temperature is supposed to pulsate equally over the entire volume. A simple analysis [4] shows that the frequency should not exceed $\sqrt{\nu_{max}} = l/m \cdot \lambda_1/c'$. The lower frequency limit is dictated by the end effects on the probe pulsation amplitude. These conditions set the frequency range from measuring the thermal activity of liquids from 0.1 Hz to 10 kHz. Temperature waves transmitted from the probe into the medium attenuate fast, depending on the frequency. At frequencies below 100 Hz the temperature waves die out almost completely after having traveled a distance of approximately 0.1 mm. The ratio of temperature wave amplitudes is determined according to the formula $|\Theta_{r_1}/\Theta_{r_0}| = h_0(r_1\sqrt{\omega/a})/h_0(r_0\sqrt{\omega/a})$. The argument in the numerator (at $r_1 = 0.5$ mm) is 50 times greater than the argument in the denominator (at $r_0 = 10\mu$m), which, according to the data tabulated in [5], makes the amplitude decrease to a fraction smaller than 1/1000. Thus, under the test conditions chosen here, the effect of the capillary wall on the temperature field and on the amplitude of probe temperature pulsations is negligibly small.

Principle of the Compensation Circuit of Measuring the Thermal Activity of Liquids. We will now determine the voltage drop across the probe carrying current $I = J_0 + Je^{i\omega t}$ while its temperature pulsates $\Theta = |\Theta|e^{i(\omega t-\varphi)}$, where $|\Theta|$ and φ are defined by Eqs. (2). The probe impedance is

$$Z = R + R\alpha|\Theta|\exp[i(\omega t - \varphi)]. \tag{3}$$

Therefore,

$$U = IZ = [RJ + J_0R\alpha|\Theta|e^{i\varphi}]e^{i\omega t}. \tag{4}$$

We have omitted here the dc as well as the 2ω double-frequency voltage component, since all measurements are made using the narrow-band amplifier tuned to the frequency ω. Besides, the second harmonic is much smaller than the fundamental component, since $J \ll J_0$.

It is evident from expression (4) that the amplitude of the probe voltage is a complex quantity with the reactive component $U_1 = J_0R_\alpha|\Theta|\sin\varphi$. The equivalent ac impedance of the probe in (4) can be expressed as

$$Z^e = \frac{U}{I} = R + \frac{J_0}{J}R\alpha|\Theta|\cos\varphi - i\frac{J_0}{J}R\alpha|\Theta|\sin\varphi. \tag{5}$$

Similarly, the impedance of resistor R shunted by a capacitance C_e is

$$Z_{RC} = R - i\omega R^2C_e, \tag{6}$$

since in our case $(J/J_0)R\alpha|\Theta|\cos\varphi \ll R$ and $(\omega RC_e)^2 \ll 1$. Equating (6) and (5), we obtain

$$\frac{\omega^2 mcRC_e}{2J_0^2\alpha R} = \frac{|\Theta|}{\Theta_0}\sin\varphi. \tag{7}$$

For this reason, the thermal activity of liquids may be determined by means of the bridge circuit where the appropriate resistance is shunted by a variable capacitance (see Fig. 2), but it is necessary that $RC_e = R_2C_2 = R_4C_4$.

With the aid of relations (2) and (7) we obtain the final formula for determining the thermal activity of liquids:

$$\gamma = \frac{2\pi^2 mc}{J_0^2\alpha} \cdot \frac{R_3C_2}{R_4} = \frac{1 + 2\dfrac{h_1}{h_0} \cdot \dfrac{\varkappa}{b}\sin\left(\eta_0 - \eta_1 - \dfrac{\pi}{4}\right)}{1 + 4\dfrac{\varkappa}{b} \cdot \dfrac{h_1}{h_0}\sin\left(\eta_0 - \eta_1 - \dfrac{\pi}{4}\right) + 4\left(\dfrac{\varkappa}{b}\right)^2\left(\dfrac{h_1}{h_0}\right)^2}. \tag{8}$$

TABLE 1. Results of Thermal Activity Measurements ($W/m^2 \cdot deg \cdot sec^{1/2}$) on Some Liquids at Various Frequencies (Method of heating by a current with both direct and alternating component)

Substance	Tabulated values, $\varkappa \cdot 10^4$	Test values, $\varkappa \cdot 10^4$ at frequencies (Hz)					
		20	35	70	120	170	400
Toluene	4,396	4,396	4,396	4,396	4,396	4,396	4,396
n-Heptane	4,497	4,271	4,271	4,312	4,245	4,271	4,421
n-Hexane	4,312	4,229	4,229	4,187	4,187	4,229	4,396
Carbon tetrachloride	3,705	3,793	3,751	3,777	3,787	3,688	3,939
Water	15,93	15,69	15,83	15,63	15,39	15,50	16,71

Thus, determining the thermal activity $\varkappa$ of a liquid reduces to measuring the quantities C_2, R_3/R_4, and J_0. In addition, the exact geometrical dimensions of the probe must be known as well as its density, specific heat, and temperature coefficient of resistance α. Although these quantities can in principle, be determined by test, the authors consider it more reasonable to use this method as a relative one, i.e., to find the values of b and mc'/α in the formula from calibration measurements on a liquid whose thermal activity is known. Analytical grade toluene was used by the authors as the standard substance.

Unlike the plain ac heating method, this one does not require a measurment of the alternating current amplitude and the overall measurement accuracy is thus undoubtedly improved. Since the electrical capacitance C depends very much on the frequency ($C \sim \nu^{-2}$), however, this method is not suitable for determining the thermal activity of liquids at high frequencies. The thermal activity of liquids can be measured, on the other hand, when there are rather wide temperature differences (10-15°C) across the layer thickness. This is so, because in our method the constant power term is much greater than in the plain ac heating method.

Results of Measurements. The test results are shown and compared with theoretical curves in Fig. 3. The latter have been determined for three values of the specific heats ratio $c_p\rho/c'_p\rho'$: 0.5, 0.6, and 1.52. For most measured organic substances this ratio lies between 0.5 and 0.6. The value 1.52 corresponds to water. As can be seen from these comparisons, the test points at low frequencies agree within 3% with the theoretical curve, while at high frequencies the dispersion of test values increases up to 5%. As the frequency becomes higher, the equivalent capacitance C decreases sharply and, therefore, the accuracy of the thermal activity determination becomes worse. Analytical grade and cp organic liquids were tested by the authors. Inasmuch as the purpose here was only to establish the feasibility of determining the thermal activity of liquids and to verify the assumptions on which calculation formulas had been derived, these substances were not made extra pure.

Numerical values of the thermal activity are given in Table 1 for some organic liquids at 26°C, obtained by this method at various frequencies from 20 to 400 Hz and compared with values given by other authors.

NOTATION

$\varkappa$	is the thermal activity of liquid;
λ	is the thermal conductivity of liquid;
c_p	is the specific heat of liquid;
ρ	is the density of liquid;
a	is the thermal diffusivity of liquid;
$b = c'_p\rho' r_0\sqrt{\omega}$, $\eta_0 - \eta_1$, h_1/h_0	are the modified Bessel functions whose values are tabulated in [5] for various arguments $z_0 = r_0\sqrt{\omega/a}$;
$\Theta_0 = W/c'_p m\omega$	is the amplitude of temperature pulsations due solely to the thermal capacitance of the probe;
W	is the ac power;
mc'_p	is the thermal capacitance of probe;
ω	is the frequency of temperature pulsations;
l	is the length of probe;
m	is the mass of probe;
c'_p	is the specific heat of probe;

λ' is the thermal conductivity of probe;

$\tau(r)$ is the steady-state temperature field produced by the dc component of the ac power;

$H_0^{(1)}(z)$ is the modified zeroth-order Bessel function of the argument $z = ir\sqrt{i\omega/a}$.

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